Addition and Subtraction of Radicals

1. $\sqrt[3]{48}+2\sqrt[3]{6}-5\sqrt[6]{36}=\sqrt[3]{2^4\cdot 3}+2\sqrt[3]{2\cdot 3}-5\sqrt[6]{2^2\cdot 3^2}=2\sqrt[3]{2\cdot 3}+2\sqrt[3]{2\cdot 3}-5\sqrt[6]{2^2\cdot 3^2}$

$5\sqrt[6]{2^2\cdot 3^2}\Rightarrow 5\left(2^2\cdot 3^2\right)^{\frac{1}{6}}\Rightarrow 5\cdot 2^{\frac{2}{6}}\cdot 3^{\frac{2}{6}}\Rightarrow 5\cdot 2^{\frac{1}{3}}\cdot 3^{\frac{1}{3}}\Rightarrow 5\sqrt[3]{2\cdot 3}$

$=2\sqrt[3]{2\cdot 3}+2\sqrt[3]{2\cdot 3}-5\sqrt[3]{2\cdot 3}=-\sqrt[3]{2\cdot 3}$

2. $x\sqrt{11}-2y\sqrt{11}+6\sqrt{11}=x\sqrt{11}-2y\sqrt{11}+6\sqrt{11}$

or $\left(x-2y+6\right)\sqrt{11}$

3. $\sqrt[4]{243}-2\sqrt[4]{3}=\sqrt[4]{3^5}-2\sqrt[4]{3}=3\sqrt[4]{3}-2\sqrt[4]{3}=\sqrt[4]{3}$

4. $3x\sqrt{16x^3}-2x^2\sqrt{x}+\sqrt[6]{x^3}=3x\sqrt{2^4x^3}-2x^2\sqrt{x}+\sqrt[6]{x^3}=3\cdot 2^2x^2\sqrt{x}-2x^2\sqrt{x}+\sqrt[6]{x^3}$

$\sqrt[6]{x^3}=(x^3)^{\frac{1}{6}}=x^{\frac{3}{6}}=x^{\frac{1}{2}}=\sqrt{x}$

$12x^2\sqrt{x}-2x^2\sqrt{x}+\sqrt{x}=10x^2\sqrt{x}+\sqrt{x}$

or $(10x^2+1)\sqrt{x}$

5. $5\sqrt[3]{64x^5y}-3\sqrt[3]{27x^2y}=5\sqrt[3]{2^6x^5y}-3\sqrt[3]{3^3x^2y}=5\cdot 2^2x\sqrt{x^2y}-3\cdot 3\sqrt{x^2y}=20x\sqrt{x^2y}-9\sqrt{x^2y}$

or $(20x-9)\sqrt{x^2y}$

6. $\sqrt[4]{80}+\sqrt[4]{\dfrac{1}{125}}-\sqrt[8]{25}=\sqrt[4]{2^4\cdot 5}+\sqrt[4]{\dfrac{1}{5^3}}-\sqrt[8]{5^2}=\sqrt[4]{2^4\cdot 5}+\dfrac{1}{\sqrt[4]{5^3}}-\sqrt[8]{5^2}=$

$\sqrt[8]{5^2}=(5^2)^{\frac{1}{8}}=5^{\frac{2}{8}}=5^{\frac{1}{4}}=\sqrt[4]{5}$

$\sqrt[4]{2^4\cdot 5}+\dfrac{1}{\sqrt[4]{5^3}}\cdot\dfrac{\sqrt[4]{5}}{\sqrt[4]{5}}-\sqrt[4]{5}=2\sqrt[4]{5}+\dfrac{\sqrt[4]{5}}{5}-\sqrt[4]{5}=\dfrac{10\sqrt[4]{5}+\sqrt[4]{5}-5\sqrt[4]{5}}{5}=\dfrac{6\sqrt[4]{5}}{5}$

7. $\sqrt[3]{250x^5y}-\sqrt[6]{256x^4y^2}+\sqrt[3]{54x^8y}=\sqrt[3]{2\cdot 5^3x^5y}-\sqrt[6]{2^8x^4y^2}+\sqrt[3]{2\cdot 3^3x^8y}=$

$\sqrt[6]{2^8x^4y^2}=(2^8x^4y^2)^{\frac{1}{6}}=2^{\frac{8}{6}}x^{\frac{4}{6}}y^{\frac{2}{6}}=2^{\frac{4}{3}}x^{\frac{2}{3}}y^{\frac{1}{3}}=\sqrt[3]{2^4x^2y}$

$\sqrt[3]{2\cdot 5^3x^5y}-\sqrt[3]{2^4x^2y}+\sqrt[3]{2\cdot 3^3x^8y}=5x\sqrt[3]{2x^2y}-2\sqrt[3]{2x^2y}+3x^2\sqrt[3]{2x^2y}$

or $(5x-2+3x^2)\sqrt[3]{2x^2y}$

8. $\sqrt[5]{64x^{-3}}+\sqrt[5]{-180x^{7}}=\sqrt[5]{\dfrac{2^{6}}{x^{3}}}+\sqrt[5]{-2^{2}\cdot 3^{2}\cdot 5x^{7}}=\dfrac{\sqrt[5]{2^{6}}}{\sqrt[5]{x^{3}}}\cdot\dfrac{\sqrt[5]{x^{2}}}{\sqrt[5]{x^{2}}}+\sqrt[5]{-2^{2}\cdot 3^{2}\cdot 5x^{7}}=$
$\dfrac{2\sqrt[5]{2x^{2}}}{5}-x\sqrt[5]{2^{2}\cdot 3^{2}\cdot 5x^{2}}$

9. $x\sqrt{x}-\sqrt{4x^{3}}+\sqrt{9x}=x\sqrt{x}-\sqrt{2^{2}x^{3}}+\sqrt{3^{2}x}=x\sqrt{x}-2x\sqrt{x}+3\sqrt{x}=-x\sqrt{x}+\sqrt{x}$
or $(-x+1)\sqrt{x}$

10. $\sqrt[3]{5}-\sqrt[3]{40}+3\sqrt[6]{25}=\sqrt[3]{5}-\sqrt[3]{2^{3}\cdot 5}+3\sqrt[6]{5^{2}}=\sqrt[3]{5}-2\sqrt[3]{5}+3\sqrt[6]{5^{2}}$
$\sqrt[6]{5^{2}}=(5^{2})^{\frac{1}{6}}=5^{\frac{2}{6}}=5^{\frac{1}{3}}=\sqrt[3]{5}$
$\sqrt[3]{5}-2\sqrt[3]{5}+3\sqrt[3]{5}=2\sqrt[3]{5}$

11. $\sqrt{\dfrac{4}{3}}+\sqrt{75}-\sqrt{\dfrac{12}{25}}=\dfrac{\sqrt{2^{2}}}{\sqrt{3}}+\sqrt{3\cdot 5^{2}}-\dfrac{\sqrt{2^{2}\cdot 3}}{\sqrt{5^{2}}}=\dfrac{\sqrt{2^{2}}}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}+\sqrt{3\cdot 5^{2}}-\dfrac{\sqrt{2^{2}\cdot 3}}{\sqrt{5^{2}}}=$
$\dfrac{2\sqrt{3}}{3}+5\sqrt{3}-\dfrac{2\sqrt{3}}{5}=\dfrac{10\sqrt{3}+75\sqrt{3}-6\sqrt{3}}{15}=\dfrac{79\sqrt{3}}{15}$

12. $3\sqrt{\dfrac{1}{7}}+\sqrt{112}+\sqrt{63}=3\dfrac{\sqrt{1}}{\sqrt{7}}+\sqrt{2^{4}\cdot 7}+\sqrt{3^{2}\cdot 7}=3\dfrac{\sqrt{1}}{\sqrt{7}}\cdot\dfrac{\sqrt{7}}{\sqrt{7}}+\sqrt{2^{4}\cdot 7}+\sqrt{3^{2}\cdot 7}=$
$\dfrac{3\sqrt{7}}{7}+2^{2}\sqrt{7}+3\sqrt{7}=\dfrac{3\sqrt{7}}{7}+4\sqrt{7}+3\sqrt{7}=\dfrac{3\sqrt{7}+28\sqrt{7}+21\sqrt{7}}{7}=\dfrac{52\sqrt{7}}{7}$

13. $2\sqrt[3]{125x^{4}y}-5\sqrt[3]{27xy^{4}}=2\sqrt[3]{5^{3}x^{4}y}-5\sqrt[3]{3^{3}xy^{4}}=2\cdot 5\cdot x\sqrt[3]{xy}-5\cdot 3\cdot y\sqrt[3]{xy}=$
$10x\sqrt[3]{xy}-15y\sqrt[3]{xy}$ or $(10x-15y)\sqrt[3]{xy}$

14. $\sqrt[4]{48}+\sqrt[4]{\dfrac{1}{27}}-\sqrt[3]{9}=\sqrt[4]{2^{4}\cdot 3}+\dfrac{\sqrt[4]{1}}{\sqrt[4]{3^{3}}}-\sqrt[3]{3^{2}}=\sqrt[4]{2^{4}\cdot 3}+\dfrac{\sqrt[4]{1}}{\sqrt[4]{3^{3}}}\cdot\dfrac{\sqrt[4]{3}}{\sqrt[4]{3}}-\sqrt[3]{3^{2}}=2\sqrt[4]{3}+\dfrac{\sqrt[4]{3}}{3}-\sqrt[3]{3^{2}}=$
$\dfrac{6\sqrt[4]{3}+\sqrt[4]{3}}{3}-\sqrt[3]{3^{2}}=\dfrac{7\sqrt[4]{3}}{3}-\sqrt[3]{3^{2}}$

15.

$$\sqrt{\frac{x}{7y}}-\frac{3}{\sqrt{7xy}}+\sqrt[4]{\frac{49y^2}{x^2}}=\frac{\sqrt{x}}{\sqrt{7y}}-\frac{3}{\sqrt{7xy}}+\sqrt[4]{\frac{7^2y^2}{x^2}}=$$

$$\sqrt[4]{\frac{7^2y^2}{x^2}}=\left(\frac{7^2y^2}{x^2}\right)^{\frac{1}{4}}=\frac{7^{\frac{2}{4}}y^{\frac{2}{4}}}{x^{\frac{2}{4}}}=\frac{7^{\frac{1}{2}}y^{\frac{1}{2}}}{x^{\frac{1}{2}}}=\frac{\sqrt{7y}}{\sqrt{x}}$$

$$\frac{\sqrt{x}}{\sqrt{7y}}-\frac{3}{\sqrt{7xy}}+\frac{\sqrt{7y}}{\sqrt{x}}=\frac{\sqrt{x}}{\sqrt{7y}}\cdot\frac{\sqrt{7y}}{\sqrt{7y}}-\frac{3}{\sqrt{7xy}}\cdot\frac{\sqrt{7xy}}{\sqrt{7xy}}+\frac{\sqrt{7y}}{\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}}=\frac{\sqrt{7xy}}{7y}-\frac{3\sqrt{7xy}}{7xy}+\frac{\sqrt{7xy}}{x}$$

$$or\ \frac{x\sqrt{7xy}-3\sqrt{7xy}+7y\sqrt{7xy}}{7xy}=\frac{(x-3+7y)\sqrt{7xy}}{7xy}$$

16. $3\sqrt[3]{24}-\sqrt[3]{81}+\sqrt[3]{54}=3\sqrt[3]{2^3\cdot 3}-\sqrt[3]{3^4}+\sqrt[3]{2\cdot 3^3}=3\cdot 2\sqrt[3]{3}-3\sqrt[3]{3}+3\sqrt[3]{2}=3\sqrt[3]{3}+3\sqrt[3]{2}$

17.

$$4\sqrt[3]{81x^4y^2}-2x\sqrt[6]{576x^8y^4}+\sqrt[3]{24x^4y^2}=4\sqrt[3]{3^4x^4y^2}-2x\sqrt[6]{2^6\cdot 3^2x^8y^4}+\sqrt[3]{2^3\cdot 3x^4y^2}=$$

$$\sqrt[6]{2^6\cdot 3^2x^8y^4}=\left(2^6\cdot 3^2x^8y^4\right)^{\frac{1}{6}}=2^{\frac{6}{6}}3^{\frac{2}{6}}x^{\frac{8}{6}}y^{\frac{4}{6}}=2^{\frac{3}{3}}3^{\frac{1}{3}}x^{\frac{4}{3}}y^{\frac{2}{3}}=\sqrt[3]{2^3\cdot 3x^4y^2}$$

$$4\sqrt[3]{3^4x^4y^2}-2x\sqrt[3]{2^3\cdot 3x^4y^2}+\sqrt[3]{2^3\cdot 3x^4y^2}=4\cdot 3\cdot x\sqrt[3]{3xy^2}-2\cdot x\cdot 2x\sqrt[3]{3xy^2}+2x\sqrt[3]{3xy^2}=$$

$$12x\sqrt[3]{3xy^2}-4x^2\sqrt[3]{3xy^2}+2x\sqrt[3]{3xy^2}=14x\sqrt[3]{3xy^2}-4x^2\sqrt[3]{3xy^2}$$

or $(14x-4x^2)\sqrt[3]{3xy^2}$

18.

$$\boxed{\begin{aligned}&\sqrt{27x^4-54x^3+27x^2}-\sqrt{75}+\sqrt{12x^4}=\sqrt{3^3x^2(x^2-2x+1)}-\sqrt{3\cdot 5^2}+\sqrt{2^2\cdot 3x^4}=\\&\sqrt{3^3x^2(x-1)^2}-\sqrt{3\cdot 5^2}+\sqrt{2^2\cdot 3x^4}=3x(x-1)-5\sqrt{3}+2x^2\sqrt{3}\end{aligned}}$$